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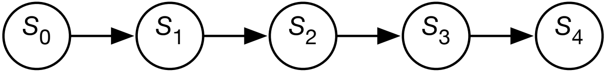

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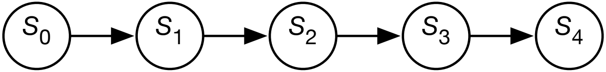

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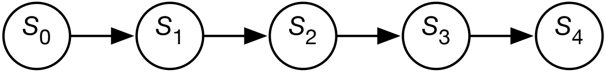

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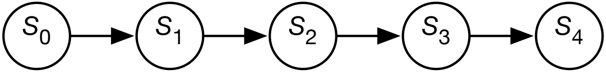

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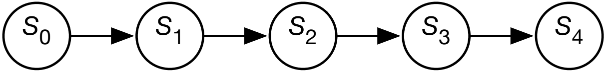

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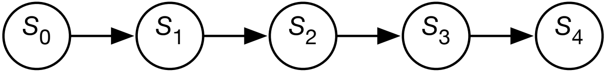

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- "The future is independent of the past given the state."

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- Under reasonable assumptions, $P(S_k)$ will approach the stationary distribution as $k \rightarrow \infty$.

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- An ergodic and aperiodic Markov chain has a unique stationary distribution P and
  $P(s) = \lim_{i\to\infty} P(S_i = s)$ — equilibrium distribution

Pagerank

Consider the Markov chain:

- Domain of S_i is the set of all web pages

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- This Markov chain converges to a stationary distribution over web pages (original $P(S_i)$ for $i = 52$ for 24 million pages and 322 million links):
  Pagerank - basis for Google's initial search engine

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Sentence: $w_1, w_2, w_3, \ldots$.

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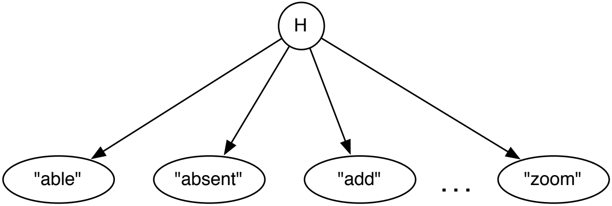

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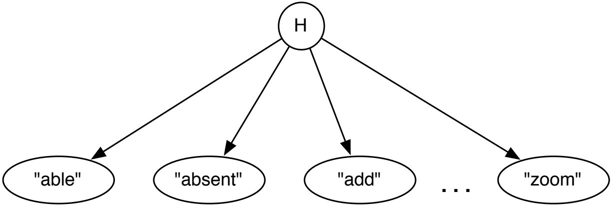

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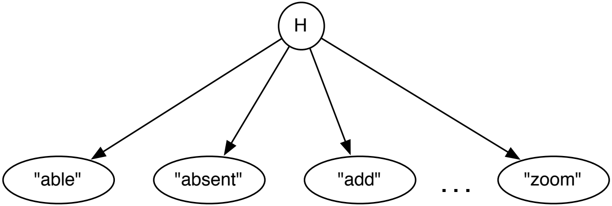

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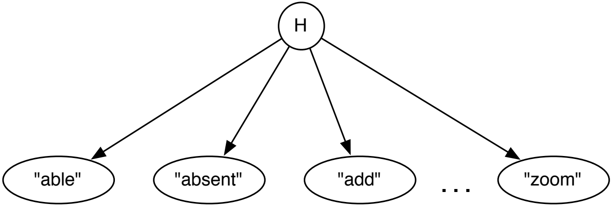

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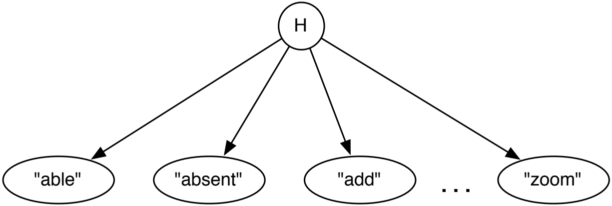

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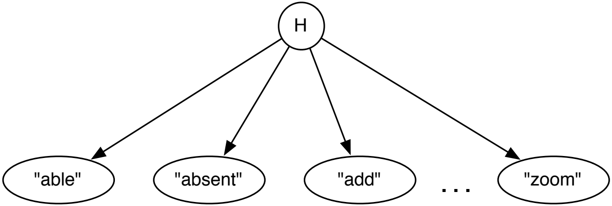

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- Given a help query: condition on the words in the query and display the most likely help page.

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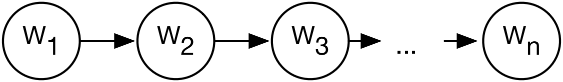

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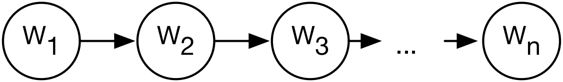

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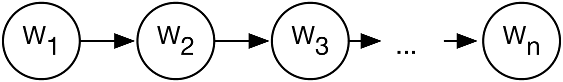

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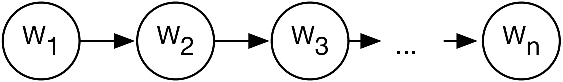

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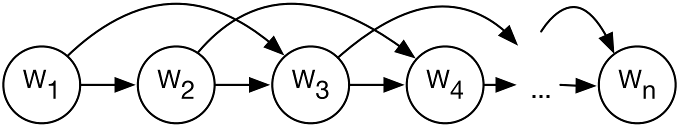

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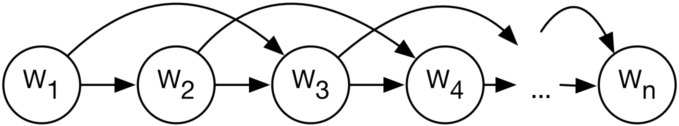

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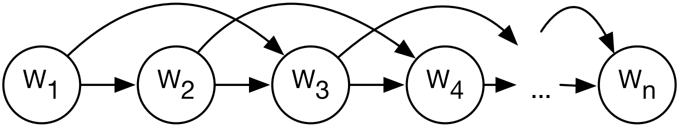

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N-gram

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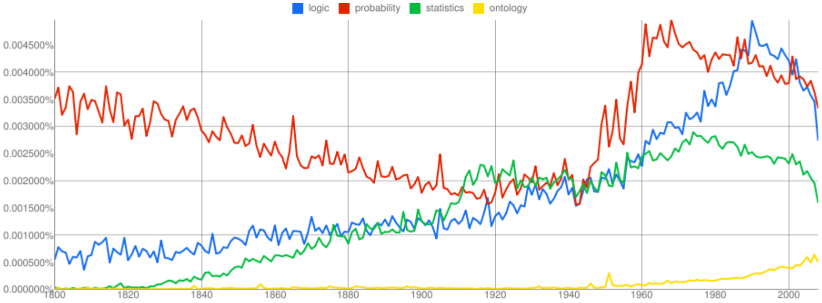
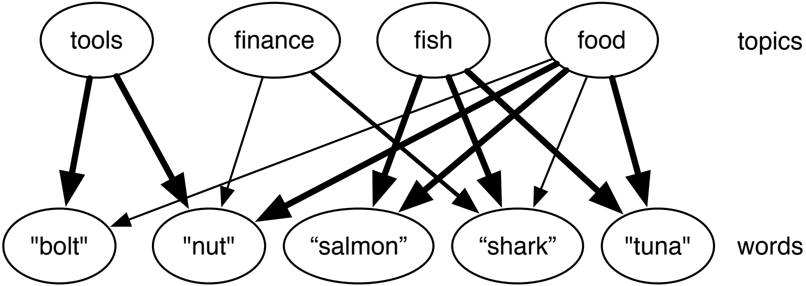
Logic, Probability, Statistics, Ontology over time

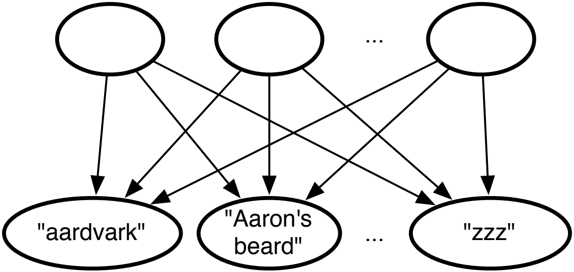


From: Google Books Ngram Viewer
(`https://books.google.com/ngrams`)

Topic Model


tools
finance
fish
food
topics
"bolt"
"nut"
“salmon”
“shark”
"tuna"
words

Google's rephil

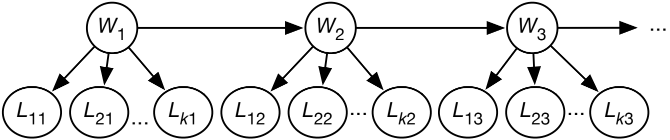


900,000 topics

350,000,000 links

12,000,000 words

Predictive Typing and Error Correction



$$domain(W_i) = \{\text{"}a\text{"}, \text{"}aarvark\text{"}, \ldots, \text{"}zzz\text{"}, \text{"}\perp\text{"}, \text{"}?\text{"}\}$$
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- *A person with a big hairy cat drank the cold milk.*
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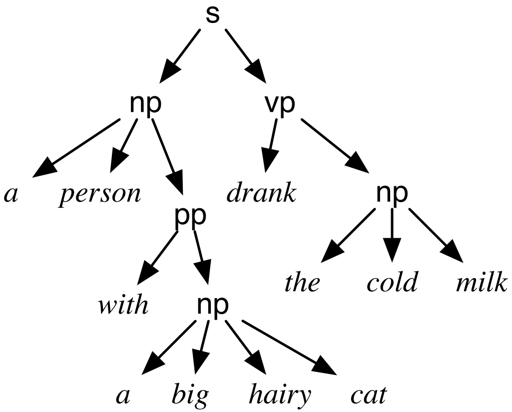

Hidden Markov Model

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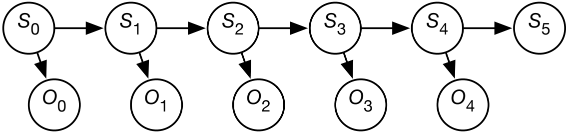

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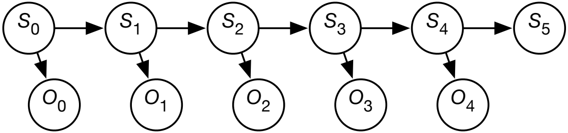

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- $P(S_0)$ specifies initial conditions
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- $P(O_i \mid S_i)$ specifies the sensor model

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What is the current belief state based on the observation history?

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What is the current belief state based on the observation history?

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- then observe O_1, query S_1. $P(S_1 \mid o_0, o_1)$

Filtering

Filtering:

$$P(S_i \mid o_0, \ldots, o_i)$$

What is the current belief state based on the observation history?

- Observe O_0, query S_0. $P(S_0 \mid o_0)$
- then observe O_1, query S_1. $P(S_1 \mid o_0, o_1)$
- then observe O_2, query S_2. $P(S_2 \mid o_0, o_1, o_2)$
- ...

Filtering

Filtering:

$$P(S_i \mid o_0, \ldots, o_i)$$

What is the current belief state based on the observation history?

- Observe O_0, query S_0. $P(S_0 \mid o_0)$
- then observe O_1, query S_1. $P(S_1 \mid o_0, o_1)$
- then observe O_2, query S_2. $P(S_2 \mid o_0, o_1, o_2)$
- ...

$$P(S_i \mid o_0, \ldots, o_i) \propto P(o_i \mid S_i o_0, \ldots, o_{i-1}) P(S_i \mid o_0, \ldots, o_{i-1})$$

Filtering

Filtering:

$$P(S_i \mid o_0, \dots, o_i)$$

What is the current belief state based on the observation history?

- Observe O_0, query S_0. $P(S_0 \mid o_0)$
- then observe O_1, query S_1. $P(S_1 \mid o_0, o_1)$
- then observe O_2, query S_2. $P(S_2 \mid o_0, o_1, o_2)$
- ...

$$P(S_i \mid o_0, \dots, o_i) \propto P(o_i \mid S_i o_0, \dots, o_{i-1}) P(S_i \mid o_0, \dots, o_{i-1})$$
$$= P(o_i \mid S_i)$$

Filtering

Filtering:

$$P(S_i \mid o_0, \ldots, o_i)$$

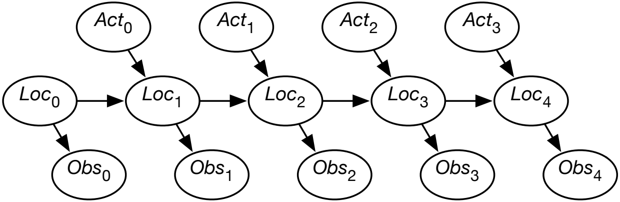
What is the current belief state based on the observation history?

- Observe O_0, query S_0. $P(S_0 \mid o_0)$
- then observe O_1, query S_1. $P(S_1 \mid o_0, o_1)$
- then observe O_2, query S_2. $P(S_2 \mid o_0, o_1, o_2)$
- ...

$$\begin{aligned} P(S_i \mid o_0, \ldots, o_i) &\propto P(o_i \mid S_i o_0, \ldots, o_{i-1}) P(S_i \mid o_0, \ldots, o_{i-1}) \\ &= P(o_i \mid S_i) \sum_{S_{i-1}} P(S_i \mid S_{i-1}) P(s_{i-1} \mid o_0, \ldots, o_{i-1}) \end{aligned}$$

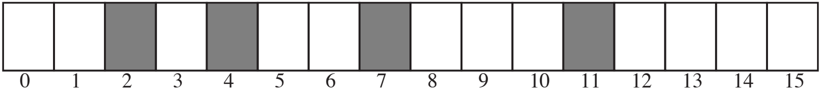
Example: localization

- Suppose a robot wants to determine its location based on its actions and its sensor readings: Localization
- This can be represented by the augmented HMM:



Example localization domain

- Circular corridor, with 16 locations:



- Doors at positions: 2, 4, 7, 11.
- Noisy Sensors
- Stochastic Dynamics
- Robot starts at an unknown location and must determine where it is.

Example Sensor Model

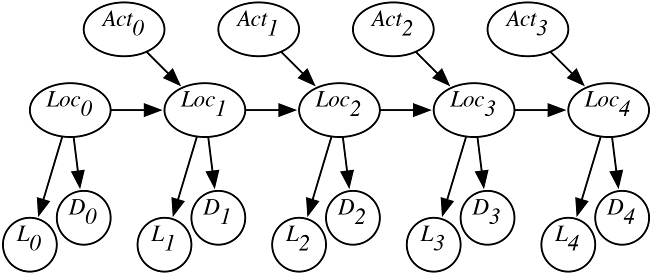
- $P(Observe\ Door \mid At\ Door) = 0.8$
- $P(Observe\ Door \mid Not\ At\ Door) = 0.1$

Example Dynamics Model

- $P(loc_{t+1} = L \mid action_t = goRight \wedge loc_t = L) = 0.1$
- $P(loc_{t+1} = L + 1 \mid action_t = goRight \wedge loc_t = L) = 0.8$
- $P(loc_{t+1} = L + 2 \mid action_t = goRight \wedge loc_t = L) = 0.074$
- $P(loc_{t+1} = L' \mid action_t = goRight \wedge loc_t = L) = 0.002$ for any other location L'.
  - All location arithmetic is modulo 16.
  - The action *goLeft* works the same but to the left.

Combining sensor information

- Example: we can combine information from a light sensor and the door sensor Sensor Fusion



S_t robot location at time t
D_t door sensor value at time t
L_t light sensor value at time t