Learning Objectives

At the end of the class you should be able to:

- recognize and represent constraint satisfaction problems
- count how big the search space is

Posing a Constraint Satisfaction Problem

A CSP is characterized by

- A set of variables $V_1, V_2, \ldots, V_n$.
- Each variable V_i has an associated domain $dom(V_i)$ which specifies the set of possible values the variable can take. (We assume domains are finite.)
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- A solution to CSP is total assignment that satisfies all the constraints.

Example: Map colouring

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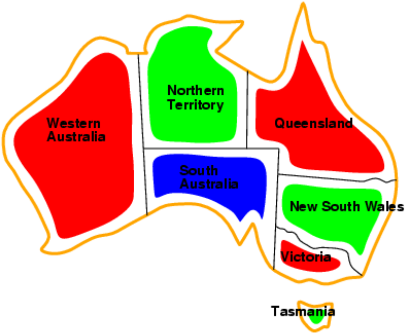
Possible solution.



Simple Examples

Example 1:

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- Domains: $\{1, 2, 3, 4\}$
- Constraints $A < B$, $B < C$

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Example 3:

- Variables: A, B, C, D, E
- Domains: $\{1, 2, 3, 4\}$
- Constraints $A < B$, $B < C, C < D, D < E$

CSP variants

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  - soft constraints specify preferences
- determine whether some property holds in all of the solutions

Example: scheduling activities

- Variables: A, B, C, D, E that represent the starting times of various activities.
- Domains: $dom(A) = \{1, 2, 3, 4\}$, $dom(B) = \{1, 2, 3, 4\}$, $dom(C) = \{1, 2, 3, 4\}$, $dom(D) = \{1, 2, 3, 4\}$, $dom(E) = \{1, 2, 3, 4\}$

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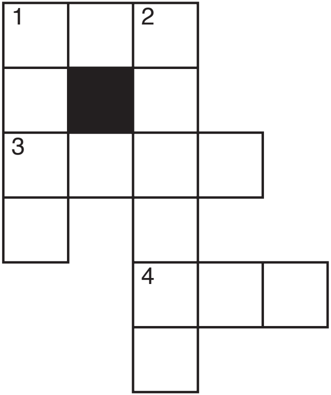
- How many total assignments are there?
- Constraints:

$$
\begin{aligned}
&(B \neq 3) \wedge (C \neq 2) \wedge (A \neq B) \wedge (B \neq C) \wedge \\
&\quad (C < D) \wedge (A = D) \wedge (E < A) \wedge (E < B) \wedge \\
&\quad (E < C) \wedge (E < D) \wedge (B \neq D).
\end{aligned}
$$

Example: Crossword Puzzle



Words:

ant, big, bus, car, has
book, buys, hold,
lane, year
beast, ginger, search,
symbol, syntax

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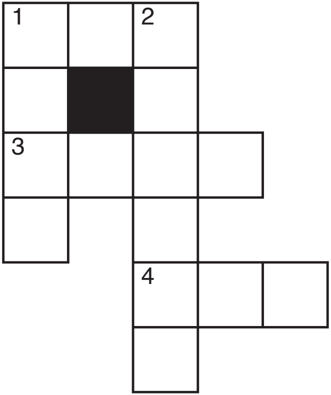

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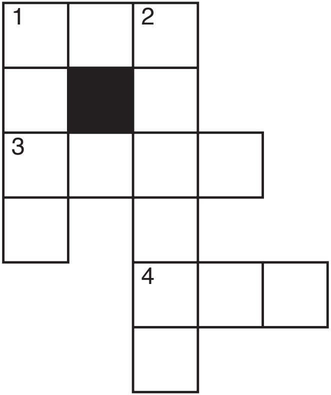

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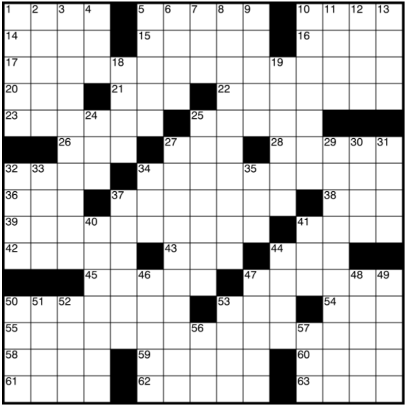

Suppose there are 10,000 words of each length (from 2 to 10).

- How many total assignments are there?

Example: Sodoku

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

- What are the variables?

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Hard and Soft Constraints

- Given a set of variables, assign a value to each variable that either
  - satisfies some set of constraints: satisfiability problems — "hard constraints"
  - minimizes some cost function, where each assignment of values to variables has some cost: optimization problems — "soft constraints"
- Many problems are a mix of hard and soft constraints (called constrained optimization problems).

Scheduling final exams

UBC exam scheduling is done by an AI system:

- 13 exam days, 52 timeslots
- 30,000 students take exams
- 1,700 sections with exams
- 105,000 student-exam pairs
- 274 rooms across 38 buildings

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- Evening courses must have evening exams

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- Preferred rooms for each exam
- Room capacities
- First-year exams on the last two days (Fall exams)
- Fourth-year exams on the last two days (Spring exams)