Learning Objectives

At the end of the class you should be able to:

- recognize and represent constraint satisfaction problems
- count how big the search space is

Posing a Constraint Satisfaction Problem

A CSP is characterized by

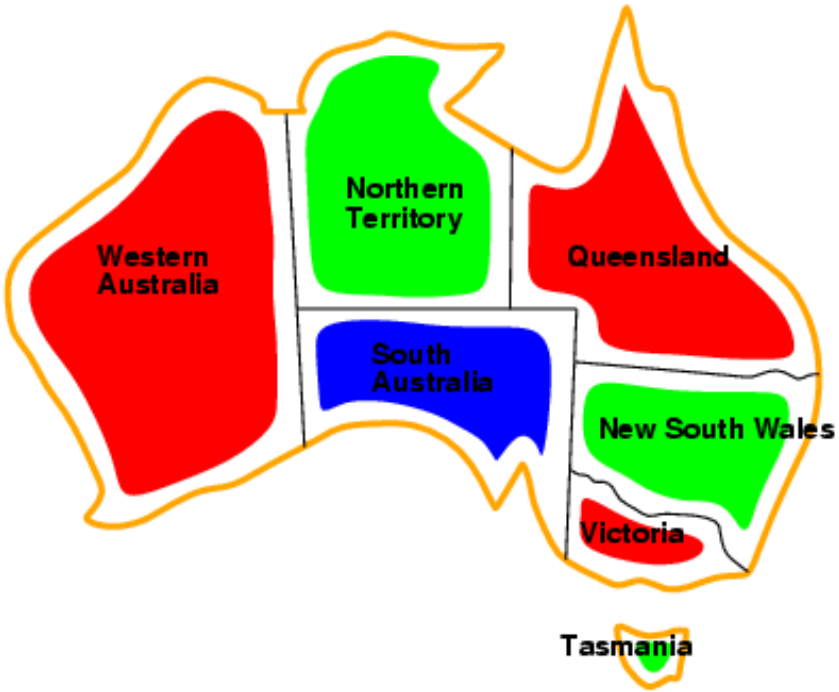
- A set of variables $V_1, V_2, \ldots, V_n$.
- Each variable V_i has an associated domain $dom(V_i)$ which specifies the set of possible values the variable can take. (We assume domains are finite.)
- A total assignment is an assignment of a value to each variable.
- A hard constraint on a subset of variables specifies which combinations of values are legal. The legal assignments are said to satisfy the constraint.
- A solution to CSP is total assignment that satisfies all the constraints.

Example: Map colouring

- Assign a colour (red, green, or blue) to each state so neighbouring states have different colours.
- What are the variables?
- What are the domains?
- How many total assignment are there?
- What are the constraints?

Example: Map colouring

Possible solution.



Simple Examples

Example 1:

- Variables: A, B, C
- Domains: $\{1, 2, 3, 4\}$
- Constraints $A < B$, $B < C$

Example 2:

- Variables: A, B, C, D
- Domains: $\{1, 2, 3, 4\}$
- Constraints $A < B$, $B < C, C < D$

Example 3:

- Variables: A, B, C, D, E
- Domains: $\{1, 2, 3, 4\}$
- Constraints $A < B$, $B < C, C < D, D < E$

CSP variants

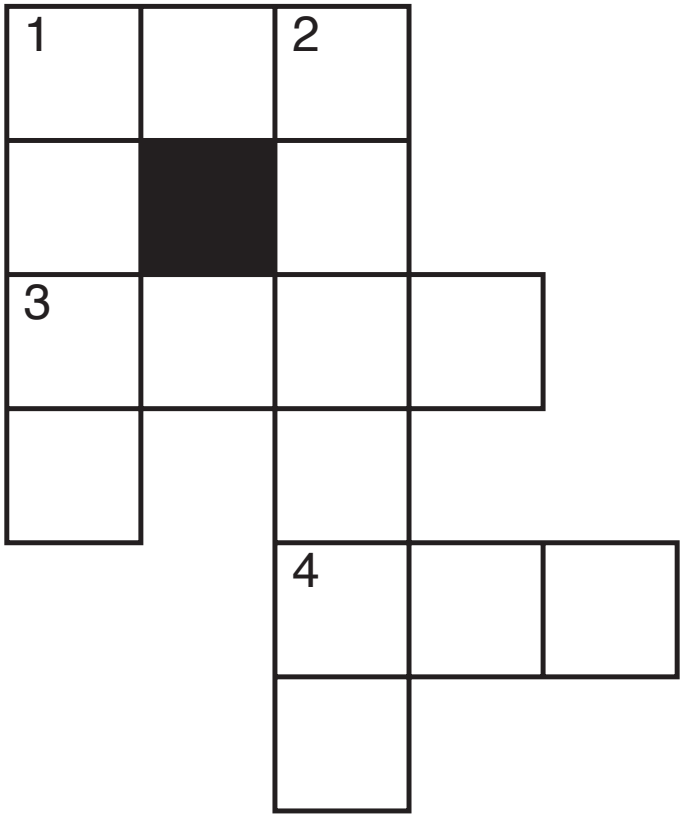
- determine whether or not a solution exists
- find a solution
- find all solutions
- count the number of solutions
- find the best solution given some solution quality
  - soft constraints specify preferences
- determine whether some property holds in all of the solutions

Example: scheduling activities

- Variables: A, B, C, D, E that represent the starting times of various activities.
- Domains: $dom(A) = \{1, 2, 3, 4\}$, $dom(B) = \{1, 2, 3, 4\}$, $dom(C) = \{1, 2, 3, 4\}$, $dom(D) = \{1, 2, 3, 4\}$, $dom(E) = \{1, 2, 3, 4\}$
- What are some total assignments?
- How many total assignments are there?
- Constraints:

$$(B \neq 3) \wedge (C \neq 2) \wedge (A \neq B) \wedge (B \neq C) \wedge (C < D) \wedge (A = D) \wedge (E < A) \wedge (E < B) \wedge (E < C) \wedge (E < D) \wedge (B \neq D).$$

Example: Crossword Puzzle

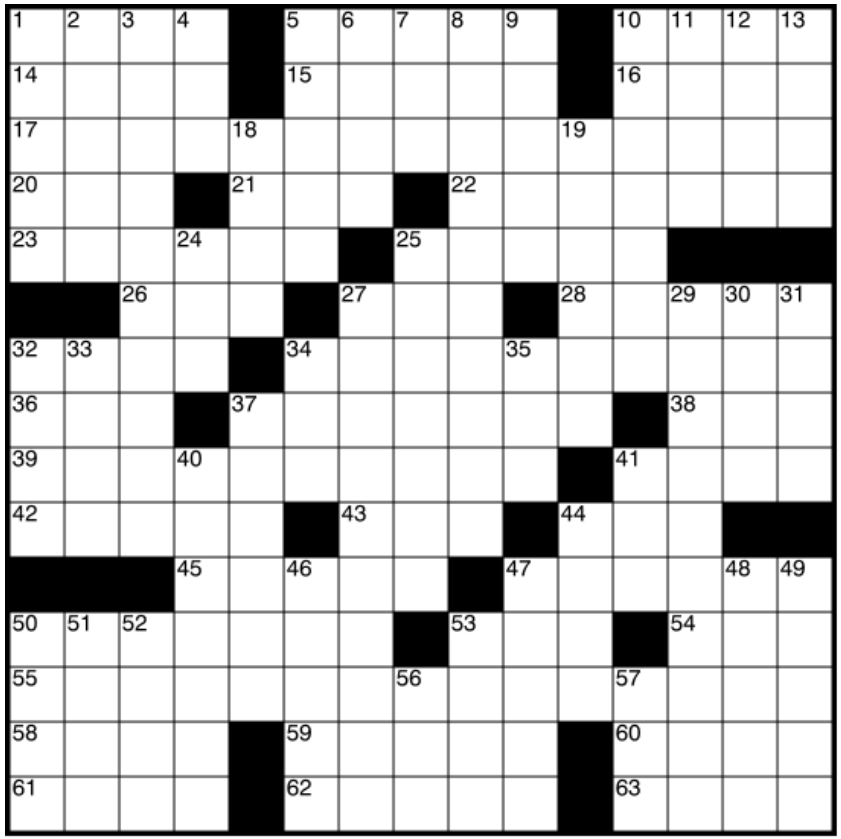


Words:

ant, big, bus, car, has
book, buys, hold,
lane, year
beast, ginger, search,
symbol, syntax

- What are the variables?
- What are their domains?
- How many total assignments are there?
- What are the constraints?

Example: Crossword Puzzle



Suppose there are 10,000 words of each length (from 2 to 10).

- How many total assignments are there?

Example: Sodoku

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

- What are the variables?
- What is their domain?
- How many total assignments are there?
- What are the constraints?

Hard and Soft Constraints

- Given a set of variables, assign a value to each variable that either
  - satisfies some set of constraints: satisfiability problems — "hard constraints"
  - minimizes some cost function, where each assignment of values to variables has some cost: optimization problems — "soft constraints"
- Many problems are a mix of hard and soft constraints (called constrained optimization problems).

Scheduling final exams

UBC exam scheduling is done by an AI system:

- 13 exam days, 52 timeslots
- 30,000 students take exams
- 1,700 sections with exams
- 105,000 student-exam pairs
- 274 rooms across 38 buildings

- What are the variables?
- What are the domains?
- How many total assignments are there?
- What are the constraints?

UBC Exam Scheduling Hard Constraints

- There can't be more than 30 conflicts for a section
- Allowable times for each exam
- Allowable rooms for each exam
- Requested room features for each exam
- Unrelated exams cannot share a room
- Cross-listed courses must have the same exam time
- Evening courses must have evening exams

UBC Exam Scheduling Soft Constraints

Try to minimize:

- Conflicts
- Students with 2+ exams on the same day
- Students with 3+ exams in 4 consecutive timeslots
- Students with back-to-back exams
- Students with less than 8 timeslots between exams
- Preferred times for each exam
- Preferred rooms for each exam
- Room capacities
- First-year exams on the last two days (Fall exams)
- Fourth-year exams on the last two days (Spring exams)