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- What assumptions are made here?

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- $P(Es)$ is a normalizing constant so the probabilities of the models sum to 1.
- You could try to fit the training data as well as possible by picking the maximum likelihood model, but that overfits.

Independent and Identically Distributed

- Examples $Es = [e_1, \ldots, e_k]$ are independent and identically distributed (i.i.d.) given model m if

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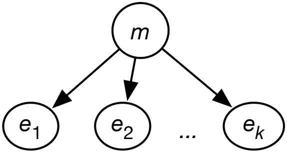

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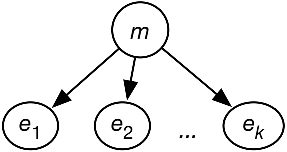

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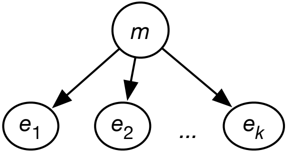
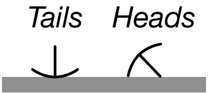


- Conditioning on the observed e_i and querying an unobserved e_j provides a probabilistic prediction for unseen examples.
- Conditioning on the observed e_i and querying m provides a distribution over models.

Learning probabilities — the simplest case

Observe tosses of thumbtack:
n_0 instances of $Heads = false$
n_1 instances of $Heads = true$
what should we use as $P(heads)$?



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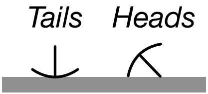

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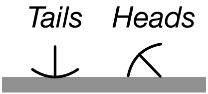

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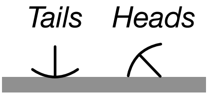

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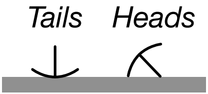

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— empirical frequency overfits to the data.

Example of Overfitting

- Consider a web site where people rate restaurants with 1 to 5 stars.
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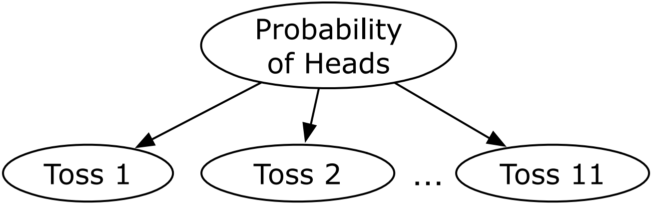
- Which restaurants have the highest average rating?
- Which restaurants have a rating of 5?
  - Only restaurants with few ratings have an average rating of 5.
- Solution: add some "average" ratings for each restaurant!

Bayesian Learning of Probabilities



aipython.org: coinTossBN in learnBayesian.py

- *Probablity_of_Heads* is a random variable representing the probability of heads.
- Domain is $\{0.0, 0.1, 0.2, \ldots, 0.9, 1.0\}$ or interval $[0, 1]$.
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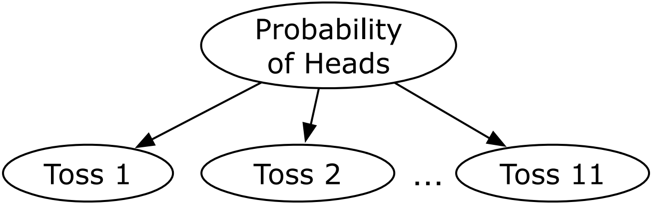

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- $Toss\#i$ is independent of $Toss\#j$ (for $i \neq j$) given *Probablity_of_Heads*
- i.i.d. or independent and identically distributed.

Bayesian Learning of Probabilities

- Y has two outcomes y and $\neg y$.
  We want the probability of y given training examples Es.
- Treat the probability of y as a real-valued random variable on the interval $[0, 1]$, called ϕ. Bayes' rule gives:

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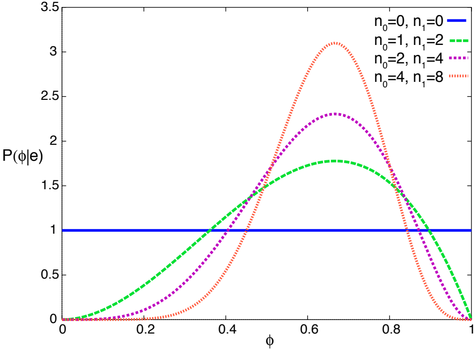
- Suppose Es is a sequence of n_1 instances of y and n_0 instances of $\neg y$:

  $$P(Es \mid \phi{=}p) = p^{n_1} \times (1 - p)^{n_0}$$

- Uniform prior: $P(\phi{=}p) = 1$ for all $p \in [0, 1]$.

Posterior Probabilities for Different Training Examples (beta distribution)



AIPython.org see probBeta.py

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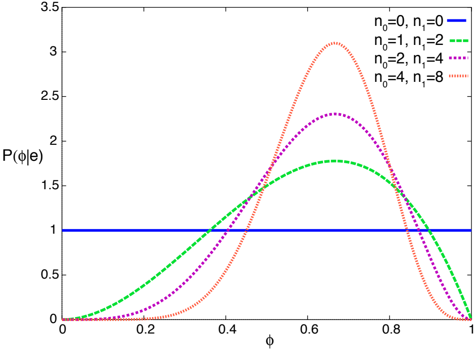

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- The mode is the empirical average.
- The expected value is Laplace smoothing (pseudo-count of 1).

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Beta Distribution

$$Beta^{\alpha_0,\alpha_1}(p) = \frac{1}{K} p^{\alpha_1 - 1} \times (1 - p)^{\alpha_0 - 1}$$

where K is a normalizing constant. $\alpha_i > 0$.

- The uniform distribution on $[0, 1]$ is $Beta^{1,1}$.
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- If the prior is of the form of a beta distribution, so is the posterior — called a conjugate distribution.

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- A Dirichlet distribution has form:

$$Dirichlet^{\alpha_1,\ldots,\alpha_k}(p_1,\ldots,p_k) = \frac{\prod_{j=1}^{k} p_j^{\alpha_j - 1}}{Z}$$

where

- p_i is the probability of the ith outcome (and so $0 \leq p_i \leq 1$)
- α_i is a positive real number (a "count")
- Z is a normalizing constant that ensures the integral over all the probability values is 1.

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- the numbers from different experts can be added
- the number can be combined with real data
- the effective sample size (m) can be tuned to reflect expertise.

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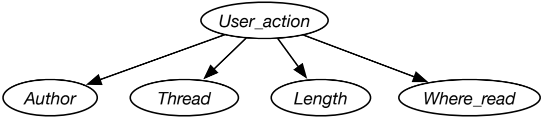
Probabilistic Classifiers

- A Bayes classifier is a probabilistic model that is used for supervised learning.
- idea: the role of a class is to predict the values of features for members of that class.
- In a naive Bayes classifier the input features are conditionally independent of each other given the classification.
  Example: Suppose an agent wants to predict the user action based on properties of an online discussion:



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Naive Bayes Classifiers

With inputs $X_1{=}v_1, \ldots, X_k{=}v_k$, and classification, Y:

$$
\begin{aligned}
& P(y \mid X_1{=}v_1, \ldots, X_k{=}v_k) \\
& \quad = \frac{P(y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid y)}{P(y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid y) + P(\neg y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid \neg y)} \\
& \quad = \frac{1}{1 + \frac{P(\neg y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid \neg y)}{P(y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid y)}} \\
& \quad = \frac{1}{1 - exp(log(\frac{P(\neg y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid \neg y)}{P(y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid y)}))} \\
& \quad = sigmoid\left(log(\frac{P(y)}{P(\neg y)}) + \sum_i \log \frac{(X_i{=}v_i \mid y)}{(X_i{=}v_i \mid \neg y)} \right)
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&\quad= \frac{1}{1 + \frac{P(\neg y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid \neg y)}{P(y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid y)}} \\
&\quad= \frac{1}{1 - exp(log(\frac{P(\neg y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid \neg y)}{P(y) * \prod_{i=1}^{k} P(X_i{=}v_i \mid y)}))} \\
&\quad= sigmoid\left(log(\frac{P(y)}{P(\neg y)}) + \sum_i \log \frac{(X_i{=}v_i \mid y)}{(X_i{=}v_i \mid \neg y)} \right)
\end{aligned}
$$

is a Logistic regression model

Naive Bayes Classifiers

With inputs $X_1{=}v_1, \ldots, X_k{=}v_k$, and classification, Y:

$$
\begin{aligned}
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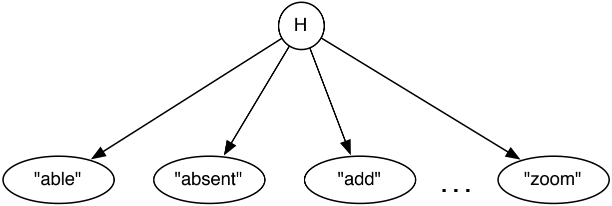
is a Logistic regression model when all X_i are observed

Naive Bayes Classifier: User's request for help



H is the help page the user is interested in.
We observe the words in the query.

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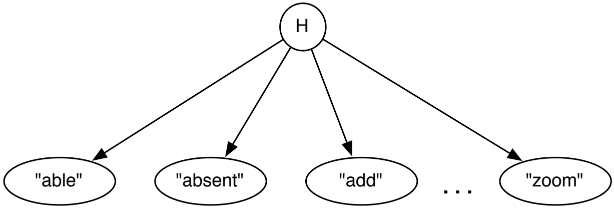

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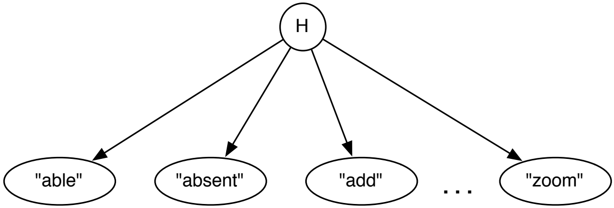

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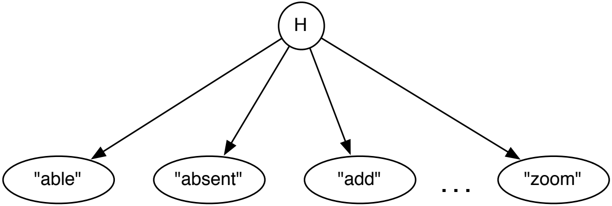

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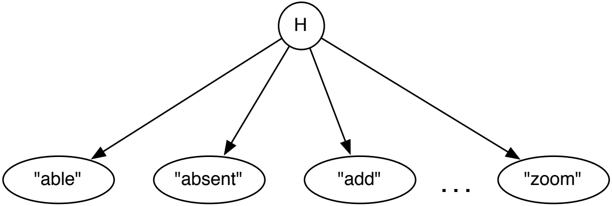

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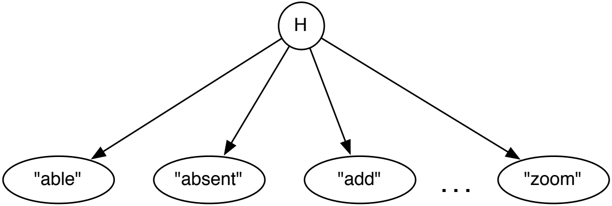

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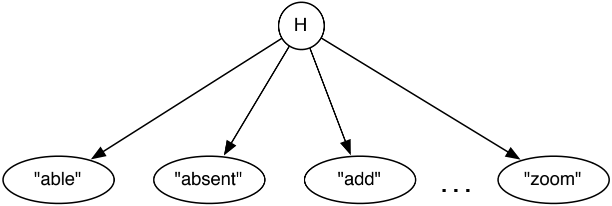

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What prior counts should be used? Can they be zero?

Help System

- Suppose the help pages are $\{h_1, \ldots, h_k\}$.
- Words are $\{w_1, \ldots, w_m\}$.
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- Maintain "counts" (pseudo counts + observed cases):
  - c_i the number of times h_i was the correct help page
  - $s = \sum_i c_i$
  - u_{ij} the number of times h_i was the correct help page and word w_j was used in the query.
- $P(h_i) = c_i / s$
- $P(w_j \mid h_i) = u_{ij} / c_i$

Learning + Inference

- Q is the set of words in the query.
- Learning: if h_i is the correct page: Increment s, c_i and u_{ij} for each $w_j \in Q$.
- Inference:

$$P(h_i \mid Q) \propto P(h_i) \prod_{w_j \in Q} P(w_j \mid h_i) \prod_{w_j \notin Q} (1 - P(w_j \mid h_i))$$

$$\left.\begin{matrix}\textit{expensive}\\ \textit{inference}\end{matrix}\right\rangle = \frac{c_i}{s} \prod_{w_j \in Q} \frac{u_{ij}}{c_i} \prod_{w_j \notin Q} \frac{c_i - u_{ij}}{c_i}$$

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expensive learning ⟩

$$= \Psi_i \prod_{w_j \in Q} \frac{u_{ij}}{c_i - u_{ij}}$$

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- What about new words?
- What do we do with new help pages?
- How can we transfer the language model to a new help system?