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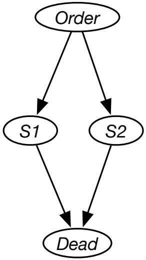
- The other case is observing then intervening.
- When the intervention is different from what actually happened, this is counterfactual reasoning, which is asking "what if something else were true".
- Let's use a more general notion of counterfactual, where you can ask "what if x were true" without knowing whether x were true.

Example: firing squad



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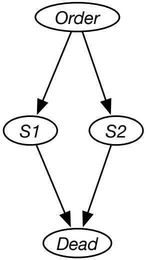

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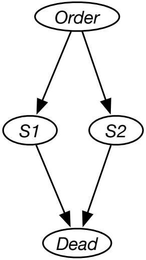

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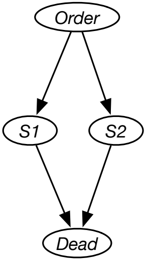

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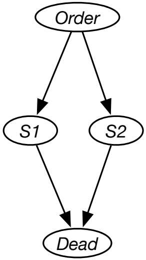

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- The prisoner is dead if any of the shooters shoot.
- One counterfactual is "if the second shooter shot, what would have happened if the second shooter had not shot?"
- Another counterfactual query is "if the prisoner died; what would have happened if shooter 2 had not shot".

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This can be implemented by constructing a causal network, from which queries from the counterfactual situation can be made.

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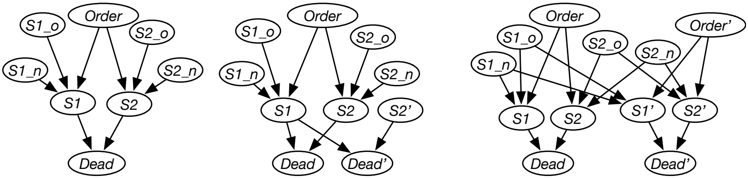
- Condition on $C'=c$
- Condition on the observations of the initial situation using unprimed variables.

Example



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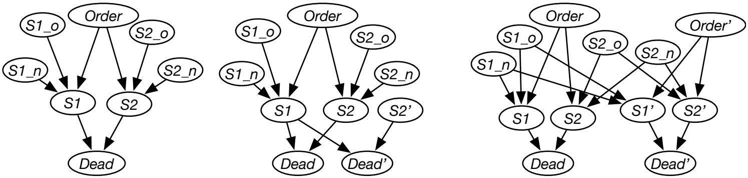

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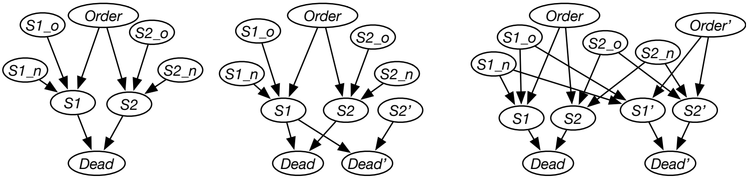

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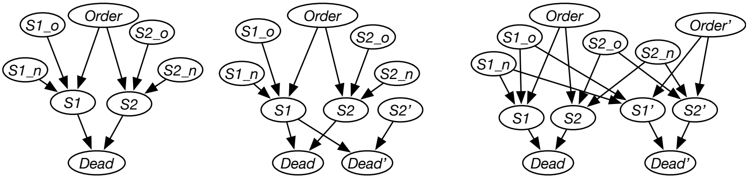

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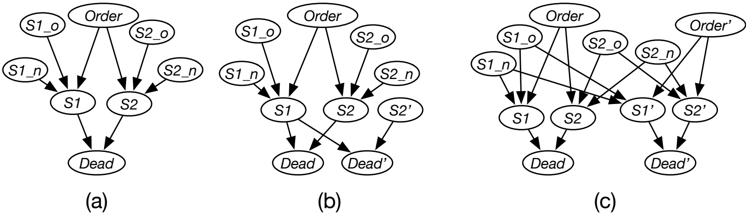

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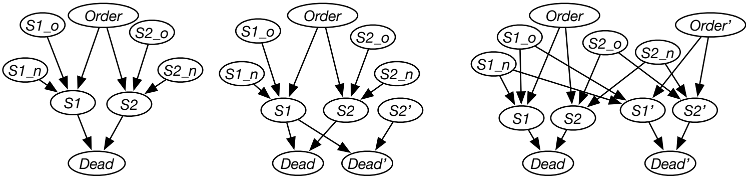

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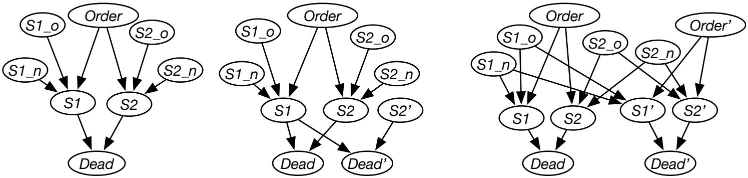

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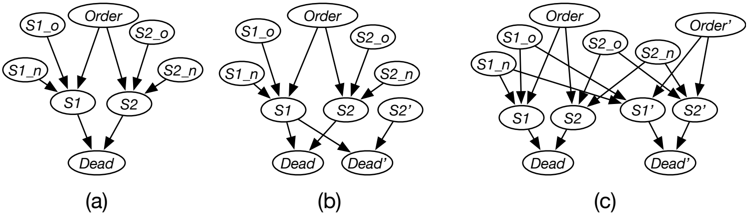

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