

At the end of the class you should be able to:

- Explain the relationship between decision-theoretic planning (MDPs) and reinforcement learning
- Implement basic state-based reinforcement learning algorithms: Q-learning

What should an agent do given:

- **Prior knowledge** possible states of the world  
possible actions
- **Observations** current state of world  
immediate reward / punishment
- **Goal** act to maximize accumulated (discounted) reward
- Like decision-theoretic planning, except model of dynamics and model of reward not given.

# Reinforcement Learning Examples

- Game - reward winning, punish losing
- Dog - reward obedience, punish destructive behavior
- Robot - reward task completion, punish dangerous behavior

- Assume there is a sequence of experiences:

*state, action, reward, state, action, reward, ....*

- The sequence of experiences up to the time the agent has to choose its action is its **history**
- The agent has to choose its action as a function of its history.
- At any time it must decide whether to do.
  - ▶ **explore** to gain more knowledge
  - ▶ **exploit** knowledge it has already discovered

# Why is reinforcement learning hard?

- What actions are responsible for a reward may have occurred a long time before the reward was received.
  - ▶ The dog is expected to determine that eating the shoe at the start of the day is what was responsible for it being scolded at the end of the day.
- The long-term effect of an action depend on what the agent will do in the future.
  - ▶ It might be okay for a robot to create a mess as long as it cleans up after itself.
- The explore-exploit dilemma: at each time should the agent be greedy or inquisitive?

# Reinforcement learning: main approaches

- search through a space of policies (controllers)
- learn a model consisting of state transition function  $P(s'|a, s)$  and reward function  $R(s, a)$ ; solve this as an MDP.
- learn  $Q^*(s, a)$ , use this to guide action.

## Recall: Asynchronous VI for MDPs, storing $Q[s, a]$

(If we knew the model:)

Initialize  $Q[S, A]$  arbitrarily

Repeat forever:

- Select state  $s$ , action  $a$
- $Q[s, a] := R(s, a) + \gamma \sum_{s'} P(s'|s, a) \left( \max_{a'} Q[s', a'] \right)$

# Asynchronous VI for Deterministic RL

initialize  $Q[S, A]$  arbitrarily

observe current state  $s$

**repeat forever:**

    select and carry out an action  $a$

    observe reward  $r$  and state  $s'$

$Q[s, a] \leftarrow r + \gamma \max_{a'} Q[s', a']$

$s := s'$



# Computing Averages: Temporal Differences

- Suppose we have a sequence of values:

$$v_1, v_2, v_3, \dots$$

and want a running estimate of the average of the first  $k$  values:

$$A_k = \frac{v_1 + \dots + v_k}{k}$$

## Temporal Differences (cont)

- Suppose we know  $A_{k-1}$  and a new value  $v_k$  arrives:

$$\begin{aligned}A_k &= \frac{v_1 + \dots + v_{k-1} + v_k}{k} \\ &= \frac{(k-1)A_{k-1} + v_k}{k}\end{aligned}$$

Let  $\alpha_k = \frac{1}{k}$ , then

$$\begin{aligned}A_k &= (1 - \alpha_k)A_{k-1} + \alpha_k v_k \\ &= A_{k-1} + \alpha_k(v_k - A_{k-1})\end{aligned}$$

“TD formula”

- Often we use this update with  $\alpha$  fixed.
- We can guarantee convergence to average if  $\sum_{k=1}^{\infty} \alpha_k = \infty$  and  $\sum_{k=1}^{\infty} \alpha_k^2 < \infty$ .
- E.g.,  $\alpha_k = 10/(9+k)$  treats more recent experiences more, but converges to average.

- **Idea:** store  $Q[\text{State}, \text{Action}]$ ; update this as in asynchronous value iteration, but using experience (empirical probabilities and rewards).
- Suppose the agent has an experience  $\langle s, a, r, s' \rangle$
- This provides one piece of data to update  $Q[s, a]$ .
- An experience  $\langle s, a, r, s' \rangle$  provides a new estimate for the value of  $Q^*(s, a)$ :

$$r + \gamma \max_{a'} Q[s', a']$$

which can be used in the TD formula giving:

$$Q[s, a] := Q[s, a] + \alpha \left( r + \gamma \max_{a'} Q[s', a'] - Q[s, a] \right)$$

initialize  $Q[S, A]$  arbitrarily

observe current state  $s$

**repeat forever:**

    select and carry out an action  $a$

    observe reward  $r$  and state  $s'$

$$Q[s, a] := Q[s, a] + \alpha (r + \gamma \max_{a'} Q[s', a'] - Q[s, a])$$

$s := s'$

# Properties of Q-learning

- Q-learning converges to an optimal policy, no matter what the agent does, as long as it tries each action in each state enough.
- But what should the agent do?
  - ▶ exploit: when in state  $s$ , select an action that maximizes  $Q[s, a]$
  - ▶ explore: select another action

# Problems with Q-learning

- It does one backup between each experience.
  - ▶ Is this appropriate for a robot interacting with the real world?
  - ▶ An agent can make better use of the data by
    - remember previous experiences and use these to update model (action replay)
    - building a model, and using MDP methods to determine optimal policy.
    - doing multi-step backups
- It learns separately for each state.