On-policy Learning

- Q-learning does off-policy learning: it learns the value of an optimal policy, no matter what it does.
- This could be bad if the exploration policy is dangerous.
- On-policy learning learns the value of the policy being followed.
  e.g., act greedily 80% of the time and act randomly 20% of the time
- Why? If the agent is actually going to explore, it may be better to optimize the actual policy it is going to do.
- SARSA uses the experience $\langle s, a, r, s', a' \rangle$ to update $Q[s, a]$.

SARSA

initialize $Q[S, A]$ arbitrarily
observe current state s
select action a
repeat forever:
  carry out action a
  observe reward r and state s'
  select action a' using a policy based on Q
  $Q[s, a] := Q[s, a] + \alpha\,(r + \gamma Q[s', a'] - Q[s, a])$
  $s := s'$
  $a := a'$

Q-learning with Action Replay

initialize $Q[S, A]$ arbitrarily
$E = \{\}$
observe current state s
select action a
repeat forever:
    carry out action a
    observe reward r and state s'
    $E := E \cup \{\langle s, a, r, s' \rangle\}$
    $Q[s, a] := Q[s, a] + \alpha \left(r + \gamma \max_{a'} Q[s', a'] - Q[s, a]\right)$
    repeat for a while:
        select $\langle s_1, a_1, r_1, s'_1 \rangle \in E$
        $Q[s_1, a_1] := Q[s_1, a_1] + \alpha \left(r_1 + \gamma \max_{a'_1} Q[s'_1, a'_1] - Q[s_1, a_1]\right)$
    $s := s'$
    $a := a'$

Model-based Reinforcement Learning

- Model-based reinforcement learning uses the experiences in a more effective manner.
- It is used when collecting experiences is expensive (e.g., in a robot or an online game); an agent can do lots of computation between each experience.
- Idea: learn the MDP and interleave acting and planning.
- After each experience, update probabilities and the reward, then do some steps of asynchronous value iteration.

Model-based learner

Data Structures: $Q[S, A]$, $T[S, A, S]$, $C[S, A]$, $R[S, A]$
Assign Q, R arbitrarily, $C = 0$, $T = 0$
observe current state s
repeat forever:
    select and carry out action a
    observe reward r and state s'
    $T[s, a, s'] := T[s, a, s'] + 1$
    $C[s, a] := C[s, a] + 1$
    $R[s, a] := R[s, a] + (r - R[s, a])/C[s, a]$
    repeat for a while:
        select state s_1, action a_1
        $$Q[s_1, a_1] := R[s_1, a_1] + \sum_{s_2} \frac{T[s_1, a_1, s_2]}{C[s_1, a_1]} \left(\gamma \max_{a_2} Q[s_2, a_2]\right)$$
    $s := s'$

What goes wrong with this?

Reinforcement Learning with Features

- Usually we don't want to reason in terms of states, but in terms of features.
- In state-based methods, information about one state cannot be used by similar states.
- If there are too many parameters to learn, it takes too long.
- Idea: Express the value (Q) function as a function of the features. Most typical is a linear function of the features, or a neural network.

Reinforcement Learning

- flat or modular or hierarchical
- explicit states or features or individuals and relations
- static or finite stage or indefinite stage or infinite stage
- fully observable or partially observable
- deterministic or stochastic dynamics
- goals or complex preferences
- single agent or multiple agents
- knowledge is given or knowledge is learned
- perfect rationality or bounded rationality

SARSA with Generalization

```
controller SARSA_with_Generalization(Learner, γ)
    Inputs
        Learner with operations Learner.add(x, y) and Learner.predict(x).
        γ ∈ [0, 1]: discount factor
    observe current state s
    select action a
    repeat
        do(a)
        observe reward r and state s'
        select action a' based on Learner.predict((s', a'))
        Learner.add((s, a), r + γ * Learner.predict((s', a')))
        s := s'
        a := a'
    until termination
```

Review: Gradient descent

To find a (local) minimum of a real-valued function $f(x)$:

- assign an arbitrary value to x
- repeat

$$x := x - \eta \frac{df}{dx}$$

where η is the step size

To find a local minimum of real-valued function $f(x_1, \ldots, x_n)$:

- assign arbitrary values to $x_1, \ldots, x_n$
- repeat:

  for each x_i

$$x_i := x_i - \eta \frac{\partial f}{\partial x_i}$$

Review: Linear Regression

- A linear function of variables $x_1, \ldots, x_n$ is of the form

$$f^{\overline{w}}(x_1, \ldots, x_n) = w_0 + w_1 x_1 + \cdots + w_n x_n$$

  $\overline{w} = \langle w_0, w_1, \ldots, w_n \rangle$ are weights. (Let $x_0 = 1$).

- Given a set E of examples.
  Example e has input $x_i = e_i$ for each i and observed value, o_e:

$$Error_E(\overline{w}) = \sum_{e \in E} (o_e - f^{\overline{w}}(e_1, \ldots, e_n))^2$$

- Minimizing the error using gradient descent, each example should update w_i using:

$$w_i := w_i - \eta \frac{\partial Error_E(\overline{w})}{\partial w_i}$$

Review: Gradient Descent for Linear Regression

Given E: set of examples over n features

each example e has inputs $(e_1, \ldots, e_n)$ and output o_e:

Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily

repeat:

For each example e in E:

let $\delta = o_e - f^{\overline{w}}(e_1, \ldots, e_n)$

For each weight w_i:

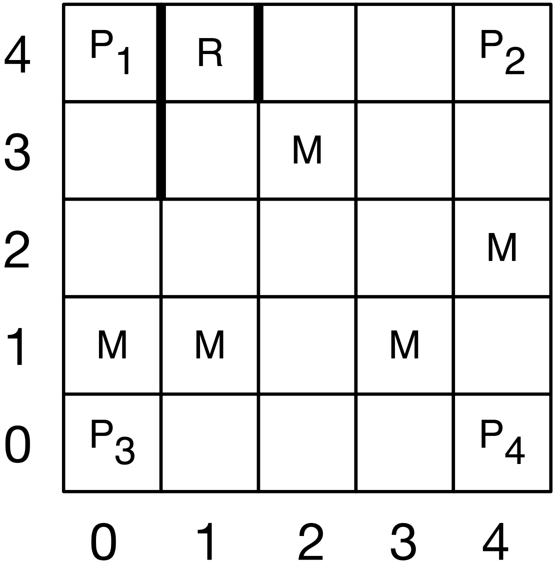
$w_i := w_i + \eta \delta e_i$

SARSA with linear function approximation

- One step backup provides the examples that can be used in a linear regression.
- Suppose $F_1, \ldots, F_n$ are the features of the state and the action.
- So $Q_{\overline{w}}(s, a) = w_0 + w_1 F_1(s, a) + \cdots + w_n F_n(s, a)$
- An experience $\langle s, a, r, s', a' \rangle$ provides the "example":
  - old predicted value: $Q_{\overline{w}}(s, a)$
  - new "observed" value: $r + \gamma Q_{\overline{w}}(s', a')$
- Treat $r + \gamma Q_{\overline{w}}(s', a')$ as a new training example for $Q(s, a)$ in linear regression (or other supervised learning algorithm).

SARSA with linear function approximation

Given γ:discount factor; η:step size
Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily
observe current state s
select action a
repeat forever:
    carry out action a
    observe reward r and state s'
    select action a' (using a policy based on $Q_{\overline{w}}$)
    let $\delta = r + \gamma Q_{\overline{w}}(s', a') - Q_{\overline{w}}(s, a)$
    For $i = 0$ to n
        $w_i := w_i + \eta \delta F_i(s, a)$
    $s := s'$
    $a := a'$



P1
R
P2
M
M
M
M
M
P3
P4
4
3
2
1
0
0
1
2
3
4

Example Features

- $F_1(s, a) = 1$ if a goes from state s into a monster location and is 0 otherwise.
- $F_2(s, a) = 1$ if a goes into a wall, is 0 otherwise.
- $F_3(s, a) = 1$ if a goes toward a prize.
- $F_4(s, a) = 1$ if the agent is damaged in state s and action a takes it toward the repair station.
- $F_5(s, a) = 1$ if the agent is damaged and action a goes into a monster location.
- $F_6(s, a) = 1$ if the agent is damaged.
- $F_7(s, a) = 1$ if the agent is not damaged.
- $F_8(s, a) = 1$ if the agent is damaged and there is a prize in direction a.
- $F_9(s, a) = 1$ if the agent is not damaged and there is a prize in direction a.

Example Features

- $F_{10}(s,a)$ is the distance from the left wall if there is a prize at location P_0, and is 0 otherwise.
- $F_{11}(s,a)$ has the value $4 - x$, where x is the horizontal position of state s if there is a prize at location P_0; otherwise is 0.
- $F_{12}(s,a)$ to $F_{29}(s,a)$ are like F_{10} and F_{11} for different combinations of the prize location and the distance from each of the four walls.
  For the case where the prize is at location P_0, the y-distance could take into account the wall.

Problems and Variants of function approximation

- This algorithm tends to overfit to current experiences. "Catastrophic forgetting".
  Solution: remember old $\langle s, a, r, s' \rangle$ experiences and to carry out some steps of action replay
- Different function approximations, such as
  - a decision tree with a linear function at the leaves (regression tree)
  - a neural network

  could be used, but they requires a representation of the states and actions.
- Use the policy to do more than one-step lookahead (better estimate of $Q(s', a')$).
  For example, compute expected value by generating samples of the rest of a game.

Evolutionary Algorithms

- In state-based MDPs and reinforcement learning, all local optima are global optima.
- With function approximation, MDP/LR algorithms can get stuck in local optima that can be arbitrarily worse that global optima
- Evolutionary algorithms can help escape local optima
- Idea:
  - maintain a population of controllers (e.g., SARSA with function approximation
  - evaluate each controller by running it in the environment
  - at each generation, the best controllers are combined to form a new population of controllers
- Performance is sensitive to representation of controller, and ways to combine them.