"Consequently he who wishes to attain to human perfection, must therefore first study Logic, next the various branches of Mathematics in their proper order, then Physics, and lastly Metaphysics."

Maimonides 1135–1204

"Logic is the beginning of wisdom, not the end."

Leonard Nimoy
"Star Trek VI: The Undiscovered Country" 1991

Individuals and Relations

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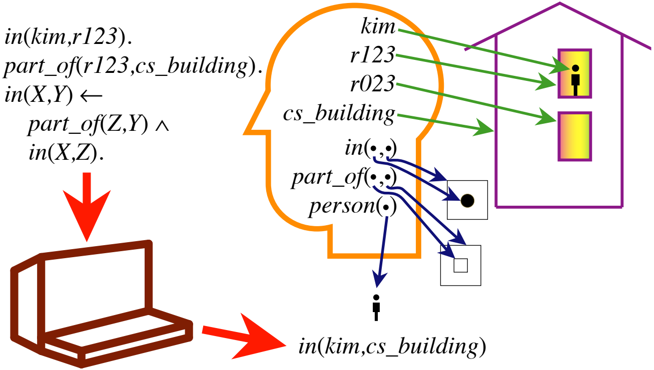
- Sometimes you may know some individual exists, but not which one.
- Sometimes there are infinitely many individuals we want to refer to (e.g., set of all integers, or the set of all stacks of blocks).

Role of Semantics in Automated Reasoning



- Users can have meanings for symbols in their head.
  They tell the computer what is true.
- The computer doesn't need to know these meanings to derive logical consequence.
- Users can interpret any answers according to their meaning.

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- atoms have structure and can include constants and logical variables
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Syntactic convention of Datalog / Prolog:

- variables start with an upper-case letter.
- constants, predicates and functions start with a lower-case letter.

In mathematics, variables typically are x, y, and z.

A Syntax of Datalog and First-order Logic

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- Quantification: $\forall$ (for all), $\exists$ (there exists)

Syntax of Datalog

- A definite clause is either an atomic symbol (a fact) or a rule of the form:

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1 \wedge \cdots \wedge b_m}_{\text{body}}$$

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- query is of the form $?b_1 \wedge \cdots \wedge b_m$.
- knowledge base is a set of definite clauses.

Example Data

The relation

Course	Section	Time	Room
cs111	7	830	dp101
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A student is busy when they have a class:

$$busy(StudentNum, Time) \leftarrow$$
$$enrolled(StudentNum, Course, Section) \land$$
$$scheduled(Course, Section, Time, Room).$$

Example Rules

$$
\begin{aligned}
& in(kim, R) \leftarrow \\
& \quad teaches(kim, cs322) \land \\
& \quad in(cs322, R). \\
& grandfather(william, X) \leftarrow \\
& \quad father(william, Y) \land \\
& \quad parent(Y, X). \\
& slithy(toves) \leftarrow \\
& \quad mimsy \land borogroves \land \\
& \quad outgrabe(mome, Raths).
\end{aligned}
$$

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A semantics specifies the meaning of sentences in the language.
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An interpretation is a triple $I = \langle D, \phi, \pi \rangle$, where

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- π is a mapping that assigns to each n-ary predicate symbol a relation: a function from D^n into $\{TRUE, FALSE\}$.

Example Interpretation

Constants: *phone*, *pencil*, *telephone*.

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- $D = \{$✂, ☎, ✎$\}$.
- $\phi(phone) =$ ☎, $\phi(pencil) =$ ✎, $\phi(telephone) =$ ☎.
- $\pi(noisy)$:

⟨✂⟩	*FALSE*	⟨☎⟩	*TRUE*	⟨✎⟩	*FALSE*

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- *D* = {✂, ☎, ✎}.
- φ(*phone*) = ☎, φ(*pencil*) = ✎, φ(*telephone*) = ☎.
- π(*noisy*):

⟨✂⟩	*FALSE*	⟨☎⟩	*TRUE*	⟨✎⟩	*FALSE*

  π(*left_of*):

⟨✂, ✂⟩	*FALSE*	⟨✂, ☎⟩	*TRUE*	⟨✂, ✎⟩	*TRUE*
⟨☎, ✂⟩	*FALSE*	⟨☎, ☎⟩	*FALSE*	⟨☎, ✎⟩	*TRUE*
⟨✎, ✂⟩	*FALSE*	⟨✎, ☎⟩	*FALSE*	⟨✎, ✎⟩	*FALSE*

Important points to note

- The domain D can contain real objects. (e.g., a person, a room, a course). D can't necessarily be stored in a computer.

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- $\pi(p)$ specifies whether the relation denoted by the n-ary predicate symbol p is true or false for each n-tuple of individuals.
- If predicate symbol p has no arguments, then $\pi(p)$ is either *TRUE* or *FALSE*.

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Ground clause $h \leftarrow b_1 \wedge \ldots \wedge b_m$ is false in interpretation I if h is false in I and each b_i is true in I, and is true in interpretation I otherwise.

Example Truths

In the interpretation given before, which of following are true?

$noisy(phone)$
$noisy(telephone)$
$noisy(pencil)$
$left\_of(phone, pencil)$
$left\_of(phone, telephone)$
$noisy(phone) \leftarrow left\_of(phone, telephone)$
$noisy(pencil) \leftarrow left\_of(phone, telephone)$
$noisy(pencil) \leftarrow left\_of(phone, pencil)$
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- If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written $KB \models g$, if g is true in every model of KB.
- That is, $KB \models g$ if there is no interpretation in which KB is true and g is false.

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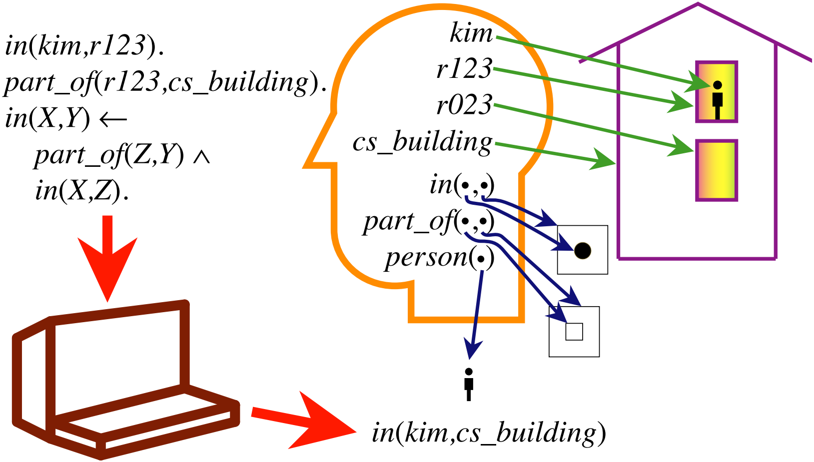
- The computer doesn't have access to the intended interpretation.
- All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
- If $KB \models g$ then g must be true in the intended interpretation.
- If $KB \not\models g$ then there is a model of KB in which g is false. As far as the computer is concerned, this could be the intended interpretation.


in(kim,r123).
part_of(r123,cs_building).
in(X,Y) ←
part_of(Z,Y) ∧
in(X,Z).
kim
r123
r023
cs_building
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part_of(•,•)
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Variables

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- Given an interpretation and a variable assignment,
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- Variables are universally quantified in the scope of a clause.
- A clause containing variables is true in an interpretation if it is true for all variable assignments.

Queries and Answers

A query is a way to ask if a body is a logical consequence of the knowledge base:

$$?b_1 \wedge \cdots \wedge b_m.$$

An answer is either

- an instance of the query that is a logical consequence of the knowledge base KB, or
- no if no instance is a logical consequence of KB.

Example Queries

$$KB = \begin{cases} in(kim, r123). \\ part\_of(r123, cs\_building). \\ in(X, Y) \leftarrow part\_of(Z, Y) \wedge in(X, Z). \end{cases}$$

Query	Answer
$?part\_of(r123, B).$	

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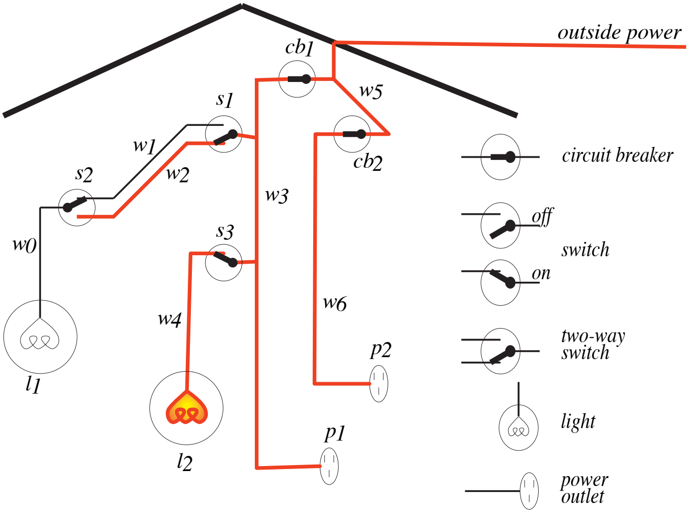
	$in(kim, cs\_building)$

Electrical Environment



Axiomatizing the Electrical Environment

% $light(L)$ is true if L is a light
$light(l_1)$. $light(l_2)$.
% $down(S)$ is true if switch S is down
$down(s_1)$. $up(s_2)$. $up(s_3)$.
% $ok(D)$ is true if D is not broken
$ok(l_1)$. $ok(l_2)$. $ok(cb_1)$. $ok(cb_2)$.

$?light(l_1)$. $\Longrightarrow$

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$?light(l_1)$. $\Longrightarrow$ *yes*
$?light(l_6)$. $\Longrightarrow$

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$?light(l_1)$. $\Longrightarrow$ yes
$?light(l_6)$. $\Longrightarrow$ no
$?up(X)$. $\Longrightarrow$ $up(s_2)$, $up(s_3)$

$connected\_to(X, Y)$ is true if component X is connected to Y

$$connected\_to(w_0, w_1) \leftarrow up(s_2).$$
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$$connected\_to(w_2, w_3) \leftarrow down(s_1).$$
$$connected\_to(w_4, w_3) \leftarrow up(s_3).$$
$$connected\_to(p_1, w_3).$$

$?connected\_to(w_0, W).$ $\Longrightarrow$

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$$connected\_to(p_1, w_3).$$

$?connected\_to(w_0, W).$ $\Longrightarrow$ $W = w_1$

$?connected\_to(w_1, W).$ $\Longrightarrow$ no

$?connected\_to(Y, w_3).$ $\Longrightarrow$

$connected\_to(X, Y)$ is true if component X is connected to Y

$$connected\_to(w_0, w_1) \leftarrow up(s_2).$$
$$connected\_to(w_0, w_2) \leftarrow down(s_2).$$
$$connected\_to(w_1, w_3) \leftarrow up(s_1).$$
$$connected\_to(w_2, w_3) \leftarrow down(s_1).$$
$$connected\_to(w_4, w_3) \leftarrow up(s_3).$$
$$connected\_to(p_1, w_3).$$

$?connected\_to(w_0, W).$ $\Longrightarrow$ $W = w_1$

$?connected\_to(w_1, W).$ $\Longrightarrow$ no

$?connected\_to(Y, w_3).$ $\Longrightarrow$ $Y = w_2,\ Y = w_4,\ Y = p_1$

$?connected\_to(X, W).$ $\Longrightarrow$

$connected\_to(X, Y)$ is true if component X is connected to Y

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$$connected\_to(p_1, w_3).$$

$?connected\_to(w_0, W).$ $\Longrightarrow$ $W = w_1$

$?connected\_to(w_1, W).$ $\Longrightarrow$ no

$?connected\_to(Y, w_3).$ $\Longrightarrow$ $Y = w_2,\ Y = w_4,\ Y = p_1$

$?connected\_to(X, W).$ $\Longrightarrow$ $X = w_0, W = w_1, \ldots$

% $lit(L)$ is true if the light L is lit

$$lit(L) \leftarrow light(L) \land ok(L) \land live(L).$$

% $live(C)$ is true if there is power coming into C

$$live(Y) \leftarrow$$
$$connected\_to(Y, Z) \land$$
$$live(Z).$$
$$live(outside).$$

This is a recursive definition of *live*.

Recursion and Mathematical Induction

$$above(X, Y) \leftarrow on(X, Y).$$
$$above(X, Y) \leftarrow on(X, Z) \wedge above(Z, Y).$$

This can be seen as:

- Recursive definition of *above*: prove *above* in terms of a base case (*on*) or a simpler instance of itself; or
- Way to prove *above* by mathematical induction: the base case is when there are no blocks between X and Y, and if you can prove *above* when there are n blocks between them, you can prove it when there are $n + 1$ blocks.

Limitations

- Suppose you had a database using the relation:

  $$enrolled(S, C)$$

  which is true when student S is enrolled in course C.
- Can you define the relation:

  $$empty\_course(C)$$

  which is true when course C has no students enrolled in it?
- Why? or Why not?

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  which is true when course C has no students enrolled in it?
- Why? or Why not?
  $empty\_course(C)$ doesn't logically follow from a set of *enrolled* relation because there are always models where someone is enrolled in a course!