Relational Learning

- flat or modular or hierarchical
- explicit states or features or individuals and relations
- static or finite stage or indefinite stage or infinite stage
- fully observable or partially observable
- deterministic or stochastic dynamics
- goals or complex preferences
- single agent or multiple agents
- knowledge is given or knowledge is learned
- perfect rationality or bounded rationality

Relational Learning

- Often the values of properties are not meaningful values but names of individuals.
- It is the properties of these individuals and their relationship to other individuals that needs to be learned.
- Relational learning has been studied under the umbrella of "Inductive Logic Programming" as the representations are often logic programs.

Example: trading agent

What does Joe like?

Individual	Property	Value
joe	*likes*	*resort_14*
joe	*dislikes*	*resort_35*
...	...	...
resort_14	*type*	*resort*
resort_14	*near*	*beach_18*
beach_18	*type*	*beach*
beach_18	*covered_in*	*ws*
ws	*type*	*sand*
ws	*color*	*white*
...	...	...

Values of properties may be meaningless names.

Example: trading agent

Possible theory that could be learned:

$$
\begin{aligned}
& prop(joe, likes, R) \leftarrow \\
& \quad prop(R, type, resort) \wedge \\
& \quad prop(R, near, B) \wedge \\
& \quad prop(B, type, beach) \wedge \\
& \quad prop(B, covered\_in, S) \wedge \\
& \quad prop(S, type, sand).
\end{aligned}
$$

Joe likes resorts that are near sandy beaches.

Inductive Logic Programming: Inputs and Output

- A is a set of atoms whose definitions the agent is learning.
- E^+ is a set of ground atoms observed true: positive examples
- E^- is the set of ground atoms observed to be false: negative examples
- B is a set of clauses: background knowledge
- H is a space of possible hypotheses. H can be the set of all logic programs defining A.

The aim is to find a simplest hypothesis $h \in H$ such that

$B \wedge h \models E^+$ and

$B \wedge h \not\models E^-$

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- $a \leftarrow b \wedge c.$
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Which is the most general? Least general?

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- For target relation $A = \{t(X_1, \ldots, X_n)\}$ what is the most general logic program?
- What is the least general logic program that is consistent with E^+ and E^-?

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- Start with a hypothesis that fits the data and keep making it simpler while still fitting the data. Initially the logic program can be E^+. Operators simplify the program, ensuring it fits the training examples.

Inductive Logic Programming: General to Specific Search

Maintain a logic program G that entails the positive examples.
Initially:

$$G = \{t(X_1, \ldots, X_n) \leftarrow\}$$

A specialization operator takes G and returns set S of clauses that specializes G. Thus $G \models S$.

Three primitive specialization operators:

- Split a clause in G on condition c. Clause $a \leftarrow b$ in G is replaced by two clauses: $a \leftarrow b \land c$ and $a \leftarrow b \land \neg c$.
- Split clause $a \leftarrow b$ on variable X, producing:

  $$a \leftarrow b \land X = t_1.$$

  $$\ldots$$

  $$a \leftarrow b \land X = t_k.$$

  where the t_i are terms.
- Remove any clause not necessary to prove the positive examples.

Top-down Inductive Logic Program

procedure $TDInductiveLogicProgram(t, B, E^+, E^-, R)$
  t: an atom whose definition is to be learned
  B: background knowledge is a logic program
  E^+: positive examples
  E^-: negative examples
  R: set of specialization operators
  Output: logic program that classifies E^+ positively and E^- negatively or $\bot$ if no program can be found
    $H \leftarrow \{t(X_1, \ldots, X_n) \leftarrow\}$
  while there is $e \in E^-$ such that $B \cup H \models e$ **do**
    if there is $r \in R$ such that $B \cup r(H) \models E^+$ **then**
      Choose $r \in R$ such that $B \cup r(H) \models E^+$
      $H \leftarrow r(H)$
    else
      return $\bot$
  return H